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Theoretical Uncertainties Associated with the Extraction of M_W at Hadron Colliders

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THEORETICAL UNCERTAINTIES ASSOCIATED WITH THE EXTRACTION OF M_W AT HADRON COLLIDERS

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In this contribution I briefly summarize several topics related to the measurement of the W-boson mass, M_W, at hadron colliders.

1 Introduction

A precise measurement of M_W, along with other measurements like the mass of the top quark, will indirectly constrain the mass of the elusive Higgs[a], the missing piece of the very successful Standard Model. This is important, an indirect measurement tells us where to look for the Higgs in direct measurement and later when (if) the Higgs is discovered a comparison of the direct and indirect measurements will provide an important test of the Standard Model. Both the Tevatron and LEP have already made very precise measurements of M_W, as reported in these proceedings[2]. In Table 1, I summarize the (CDF) expectations for the uncertainties on M_W for Run II at the Tevatron[3]. As can be seen in this table, the W production model

Table 1: Run II expectations for the uncertainties on M_W, in MeV, from Ref.[3], for an integrated luminosity of 2 fb^{-1}

Sources of Uncertainties	$W \to e\nu$	$W \to \mu\nu$
Statistical	14	20
W Production Model	30	30
Other Systematic Uncertainties	25	22
Total Uncertainty	42	40

uncertainty dominates. This is the uncertainty associated with the transverse momentum of the W, the parton distribution functions (PDF's), and the QCD and electroweak corrections. The fact that this uncertainty dominates represents both a challenge and an opportunity. It is a challenge because it is not acceptable and we should find ways to decrease this uncertainty below the experimental uncertainty. It is an opportunity because if we successfully decrease it then the Run II measurement of M_W at the Tevatron will be even better than currently anticipate. Note also that the W production model uncertainty is fully correlated between the electron and muon channels, such that not much improvement is gained by combining the two. If we succeed in controlling the W production model uncertainty we could get four (two per detectors) measurements with uncertainty smaller than 40 MeV, and an overall uncertainty of the order of 20 MeV might be possible.

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[a]currently the measurement of the weak mixing angle gives a better constraint, see Ref.[1].

In the remainder of this contribution, I review the current status of the electroweak corrections to Z and W production at hadron colliders, a ratio method to measure M_W (and Γ_W), recent developments on PDF uncertainties, and the opportunity to very precisely measure M_W at the LHC. I give my conclusions in the last section.

2 Electroweak Corrections to Z and W Production

This section is a summary of the work done in collaboration with U. Baur and W. Sakumoto in Ref.[4] (corrections to Z production) and with U. Baur and D. Wackeroth in Ref.[5] (corrections to W production). There is a shift in M_Z and M_W extracted from the data due to the electroweak corrections of the order of 100 MeV. We need to understand the uncertainty associated with that shift. The uncertainty was assumed to be of the order of 20 MeV for RunIa analysis at the Tevatron.

The electroweak corrections to Z production are also needed because the measured M_Z and Γ_z are used to calibrate the detector when compared to the values measured at LEP.

In the calculation used so far to extract M_W (Berends and Kleiss[6], 1985), only the final state photonic corrections are included using a very good approximation. The accuracy of this approximation can only be estimated by doing the full calculation. Our calculations include initial and final state corrections and their interference. We used the phase space slicing method, as in QCD[7]; the advantage of that method is that the experimental cuts can be imposed without any difficulties, without having to redo analytical integrations. In the calculations, we kept the mass of the final state charged lepton(s), it protects the final state collinear singularities. The final state photonic corrections dominate the electroweak corrections because they are enhanced by $\alpha \, log(M_{ZorW}^2/m_{lepton}^2)$ when the charged lepton and photon momentum are not recombined. These large contributions are not present in the integrated cross section as required by the KLN theorem[8]. The universal initial state collinear singularities have to be absorbed into the PDF's by factorization, in complete analogy with QCD. In principle, for the overall consistency of the calculations, the QED corrections should be added to the evolution of the PDF's and incorporated into the global fitting of PDF's. Because this has not yet been done, we only have partial information about the impact of the initial state corrections.

In the Z case, the QED corrections are gauge invariant by themselves, and so far we neglected the weak corrections, they are expected to be small. In the W case the QED corrections are not gauge invariant by themselves, the weak corrections must be included. The non trivial calculation of the matrix elements for the W case was done in Ref.[9] by D. Wackeroth and W. Hollick.

Our results are showing that, as expected, the final state corrections dominate the shape change of the distributions in the region of interest for the measurement of M_W. The most important detector effect is the recombination: when the electron and the photon are close to each other then their momenta is recombined to an effective electron momenta. This effect reduces the size of the corrections, although not to a level where they can be neglected.

The most important result is that the Z and W masses obtained by fitting with our $O(\alpha)$ calculations are about 10 MeV smaller than that obtained by fitting with the approximate calculations used so far. This is a good because this 10 MeV shift is smaller than the uncertainty so far assumed in the analysis. It is important to understand that this 10 MeV is NOT the uncertainty on the $O(\alpha)$ calculation, it is simply the difference between two calculations of the $O(\alpha)$ corrections. The uncertainty on the $O(\alpha)$ calculation can only be estimated from the size of the $O(\alpha^2)$ corrections. Now that we have shown that the approximation à la Berends and Kleiss is very good for the $O(\alpha)$ corrections, the same type of approximation could be used to obtain an estimate of the $O(\alpha^2)$ corrections.

3 Ratio Method to Measure M_W (and Γ_W)

This section is a summary of the work done in collaboration with W. Giele in Ref. [10]. Instead of using the W distribution to measure M_W and the W width, Γ_W, the ratio of W over Z distributions can be used. The normalization of the ratio should be included in the fit as it is sensitive to Γ_W. This idea is not really new, after all the measurement of M_Z and Γ_Z is already used for calibration of the detectors. However, with the upcoming high luminosity run at the Tevatron, the idea can be brought to full maturity. The main difference between the W and Z production is due to their different mass (M_V). Mass-scaled variables must therefore be used:

$$x = \frac{\mathcal{O}}{M_V}$$
$$R = \frac{\frac{d\sigma}{dx}\big|_W}{\frac{d\sigma}{dx}\big|_Z}, \tag{1}$$

where $\mathcal{O}$ is the observable under study. M_W can be fitted for such that the measured ratio R is equal to the calculated one.

The obvious limitation of the method is that it depends on the Z statistics. It is about 10 times lower than in the W case, such that the statistical uncertainty is about $\sqrt{10}$ times larger [b].

There are many advantages to the method. First, the experimental systematic uncertainties tend to cancel in the ratio. Potential problems that will spoil the cancellation are, e.g., the isolation criteria of the 2nd lepton in the Z case and some of the backgrounds that are different. Second, M_W and Γ_W are directly measured with respect to M_Z and Γ_Z which were accurately measured at LEP. Third, the QCD corrections to the ratio are smaller than for the W and Z observables themselves which means that the theoretical uncertainty on the ratio is also smaller (we have checked this statement for the transverse mass, the transverse energy of the vector boson and the transverse energy of the lepton distributions, see Ref. [10]). Finally, the expectation is that the PDF uncertainties will also be smaller.

In this ratio method, there is a clear trade-off between statistical and systematic uncertainties: the statistical uncertainty is increased while the systematic uncertainty is decreased. We therefore expect this method to be very competitive at high luminosity (Run II, TeV33 [11]) because there the standard method uncertainty is dominated by the systematic uncertainty, see Table 1.

D0 has already applied the ratio method to the transverse mass distribution with very encouraging results, see Ref. [12]. The ratio method applied to the transverse energy of the charged lepton might yield the smallest uncertainty on M_W at high luminosity.

4 Parton Distribution Function Uncertainties

This section is a summary of the work done in collaboration with W. Giele in Ref. [13]. Standard sets of PDF's do not come with uncertainties. The spread between different sets is often associated with PDF uncertainties. This is the case for the M_W analysis at the Tevatron. As is well known, it is not clear at all what this spread represents. It is time for a set of PDF's with uncertainties. In Ref. [13] we developed a method, within the framework of statistical inference, to take care of the PDF uncertainties. Here I simply explain two important steps of the method.

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[b] $\sqrt{10}$ is replaced by $\sqrt{5}$ for observables that depend on one charged lepton, such that both leptons in the Z case can be entered in the distribution.

The first step is the propagation of the uncertainty to new observables. The PDF's are assumed to be parametrized at a scale Q_0, with N parameters, $\{\lambda\} \equiv \lambda_1, \lambda_2, \ldots, \lambda_N$. The probability density distribution of these parameters, $P_{init}(\lambda)$, is also assumed to be known.

For any observable, $0(\lambda)$, the prediction is simply given by the average value over the multi-dimensional parameter space:

$$< O > = \int_V O(\lambda) P_{init}(\lambda) d\lambda. \tag{2}$$

To calculate the integral we use a Monte-Carlo approach with importance sampling. We generate 100 random sets of parameters distributed according to the initial probability density distribution, $P_{init}(\lambda)$. This corresponds to 100 sets of PDF's that represent the uncertainty. The observable can be calculated for each set, O^j, and the prediction is then given by the average value over the 100 PDF sets:

$$< O > \approx \frac{1}{100} \sum_{j=1}^{100} O^j, \tag{3}$$

whereas the PDF uncertainty is given by the standard deviation, σ_O, of the 100 PDF sets:

$$\sigma_O^2 \approx \frac{1}{100} \sum_{j=1}^{100} \left(O^j - < O >^2 \right)^2 \tag{4}$$

This gives a simple way to propagate the uncertainties to new observables, in particular there is no need for the derivative of the observable with respect to the parameters.

The second step I want to describe is the inclusion of the effect of new data on the PDF's. If the new data agrees with the prediction then the effect of the new data can be included by updating the probability density distribution with Bayes theorem. Initially, each of the 100 sets of PDF's (PDF_i) has a constant weight because of the use of importance sampling. Now each of the sets acquires a different weight given by the conditional probability density distribution of the set considering the new data:

$$P_{new}(PDF_i) = P(PDF_i/new\ data) \tag{5}$$

The latter is directly given by Bayes theorem:

$$P(PDF_i/new\ data) \propto P(new\ data/PDF_i)\ P_{init}(PDF_i) \tag{6}$$

If the uncertainties on the data are Gaussian distributed, then the weights are given by:

$$P(new\ data/PDF_i) \propto e^{-\frac{\chi_i^2}{2}} \tag{7}$$

where χ_i^2 is the chi-squared of the new data with the theory calculated with the specific set of PDF's. Prediction for yet other observables that includes the effect of the new data can now be calculated by using weighted sum. No information about the data used to derive $P_{init}(\lambda)$ is needed. Other advantages of the method are as follows. The probability density distribution of the parameters does not have to be Gaussian. A data set can be easily excluded from the fit and experimenters can include their own data into the PDF's during the analysis phase. Finally, the theory uncertainty can be easily included.

It is worth mentioning that S. Alekhin about a year ago extracted PDF's with uncertainties from deep inelastic scattering (DIS) data[14]. Both the statistical and systematic uncertainties with correlations were included. However the theoretical uncertainty was not considered. In Ref.[13] we used his results for our initial probability density distribution to predict two observables at the Tevatron: the single inclusive jet cross section and the lepton charge asymmetry in W decays. Note that the initial probability density distribution could also be entirely based on theoretical consideration, in the spirit of Bayes theorem. One remaining problem is the uncertainty associated with the choice of parametrization of the input PDF's. This is a difficult problem that does not have a clear answer and will require a compromise between the number of parameters and the smoothness of the PDF.

5   Measurement of M_W at the LHC

This section is a summary of the work done in collaboration with J. Womersley in Ref.[15]. The LHC will be a copious source of W. The cross section for W production (with appropriate cuts) at the LHC is about four times larger than at the Tevatron. The statistical uncertainty should therefore be small.

A priori, the systematic uncertainty is expected to be large at the LHC. However it is likely that the LHC will run at "low" luminosity ($\sim 10^{33} cm^{-2} s^{-1}$) for at least a year, corresponding to an integrated luminosity of $\mathcal{L} = 10 fb^{-1}$. At that luminosity the detector capabilities are very good: triggering on leptons with transverse energy as low as ~ 20 GeV is possible, the number of interactions per crossing is of the order of 2, providing a quiet environment, and the missing transverse momentum will be well measured because the hadronic calorimeters have large coverage (up to pseudorapidity of 5). Furthermore, both the ATLAS and CMS detectors offer advances over their counterparts at the Tevatron for lepton identification and measurement.

The QCD corrections to the shape of the transverse mass distribution are of the order of 10% in the region of interest. The corrections are larger than at the Tevatron ($\sim 2\%$) but still reasonable. The NNLO calculation will be useful in this case. If necessary the ratio method, explained in section 3, could be used to reduce the theoretical uncertainty.

Scaling from the current measurement at the Tevatron, about $15 \, 10^6 \; W \rightarrow e\nu$ reconstructed events are expected for $10 \; fb^{-1}$ at the LHC. The uncertainty obtained by using the parametrization developed for the Tev2000[16] study is very small, of the order of 8 MeV. It is difficult to believe that such a small uncertainty will be reached. However, we take this as an indication that there is an opportunity to make the world's best measurement of M_W, i.e. to measure M_W to a precision better than 15 MeV, the goal of TeV33.

Note also that the Bjorken-x probed is different at the LHC and the Tevatron. Therefore the PDF uncertainty will be different and from that point of view the two measurements will be complementary.

6   Conclusions: Things to do!

A precise measurement of M_W will be important to further constrain the mass of the Higgs. Current extrapolations to higher luminosity at the Tevatron indicate that the uncertainty on the

extraction of M_W will be dominated by theoretical uncertainties. We therefore have work to do to ensure that this does not remain the case. For example, the two loop corrections $(O(\alpha_s^2), O(\alpha_s\alpha),$ and $O(\alpha^2))$ are needed to evaluate the theoretical uncertainty on the one loop calculations. A more definite statement about the impact of the initial state contribution of the electroweak corrections is needed. We only have indications that they have a small effect. DIS and Tevatron data should be used to extract PDF's with uncertainties with the method described in Ref.[13]. A lot of work remains to be done but the theoretical uncertainty should be significantly decreased by the time the Tevatron takes data again.

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